Inverse Square Law (ISL)

Objectives for Tutorial on Inverse Square Law (ISL)

At the successful completion of this tutorial, one should be able to

1. Define Inverse Square Law and state the mathematical relationship between intensity and distance

2. Give 2 examples of point sources

3. Explain the importance of the ISL with respect to Radiation Protection

4. Solve mathematical problems based on the ISL

5. Explain how the Gamma Ray Dose Constant is related to the ISL

INVERSE SQUARE LAW

One of the most enduring principles of Radiation Protection is the Inverse Square Law. Physicists often tell their classes that, to minimize radiation dose, there is nothing simpler, quicker, or less expensive than simply moving away from a source. Not only is radiation dose lowered immediately, but also dramatically, as the relationship between dose and distance, like almost everything else related to radiation safety, is not linear.

A simple statement of the Inverse Square Law is that intensity varies inversely with the square of the distance from the source. It should be pointed out that the source must be a **point source**, that is, it is emitted from a small volume, and it is omnidirectional (emitted in all directions). Examples of point sources include an incandescent bulb with a small filament, the tip of a radio or TV tower, a small dot of radioactivity, or a bright object that is very far away, e.g., the sun. What is NOT a point source is a nearby 4-ft fluorescent light bulb (emitted from a large volume) or a laser beam (unidirectional since the beam is collimated).

The mathematical statement of the Inverse Square Law is displayed in the pictorial below. As indicated, the intensity decreases with the square of the distance.
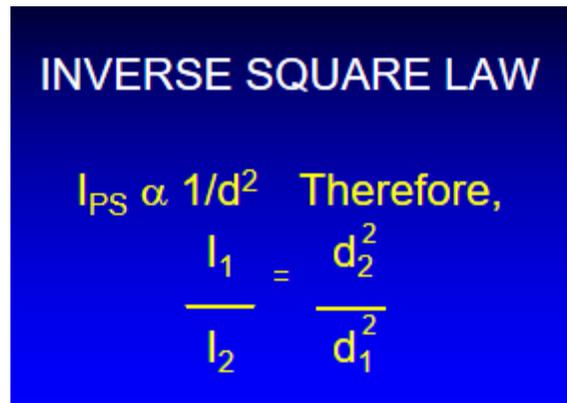

The mathematical beauty of the formula is that an intensity x its distance squared from the point source = the other intensity x its distance squared from the point source, i.e., subscripts on the left side of the equation are 1 and subscripts on the right side of the equation are 2, as illustrated below.

$$I_1 \times d_1{}^2 = I_2 \times d_2{}^2$$

The 2 similar pictorials below show a point source emitting radiation in all directions.
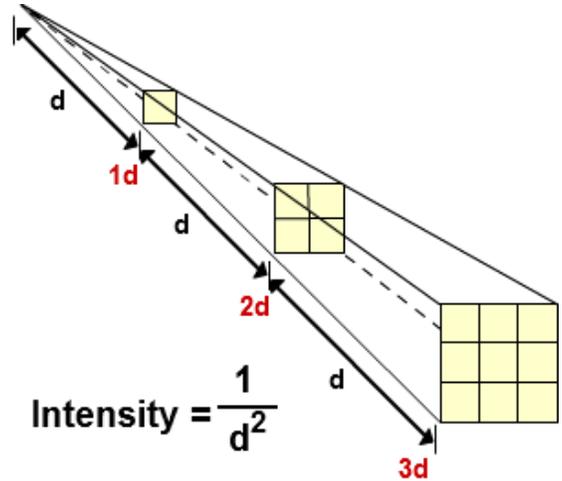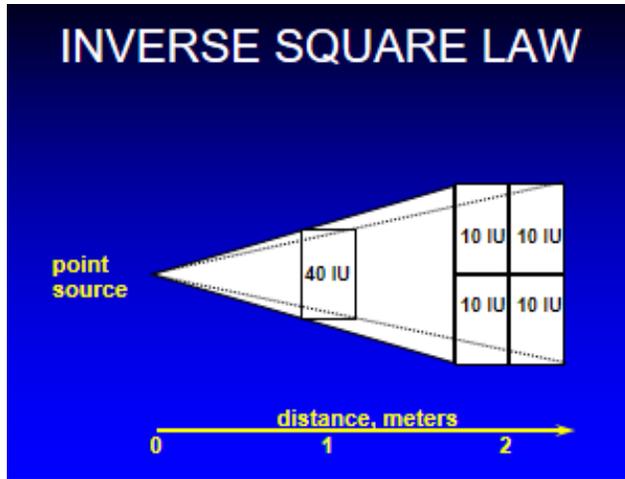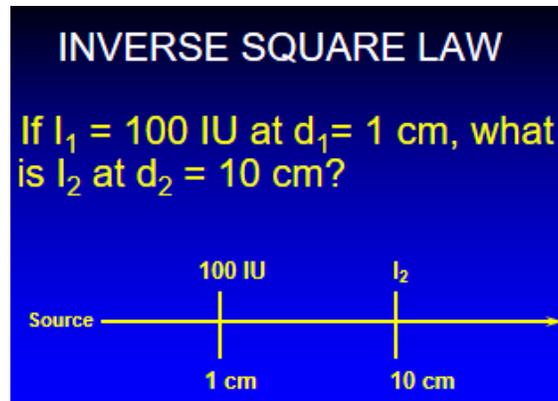
The focus of the diagram on the left is radiation moving from the left toward the right. You will observe that at 1 meter from the point source, the intensity is 40 intensity units (IU). At a distance of 2 meters, the intensity has been reduced to 10 IU in a rectangular area identical to that at 1 meter. In reality, no radiation has been lost (there is still a total of 40 IU), but it has been spread over an area 4 times as large. This is what the Inverse Square Law predicts- as we double the distance, the intensity drops to $(1/2)^2$ or 1/4 of the original value. If the distance had been tripled, the intensity would have dropped to $(1/3)^2$ or 1/9 of the original; if the distance had increased by a factor of 10, the intensity would have decreased to $(1/10)^2$ or 1/100 of the original value.

Conversely, if we move twice as close to the point source, the intensity increases by a factor of 2^2 or 4 times the original value. if we move three times as close to the point source, the intensity increases by a factor of 3^2 or 9 times of the original value; if we move ten times as close to the point source, the intensity increases by a factor of 10^2 or 100 times of the original value.

If we take a look at the sample problem below, in which we have been given I_1, d_1, and d_2, it is easy to set up the problem using the formula $I_1 \times d_1{}^2 = I_2 \times d_2{}^2$

INVERSE SQUARE LAW

If $I_1 = 100$ IU at $d_1 = 1$ cm, what is I_2 at $d_2 = 10$ cm?

100 IU $\qquad$ I_2

Source ——————|——————|——————→

1 cm $\qquad$ 10 cm

Setup and solution are shown below:

INVERSE SQUARE LAW

If $I_1 = 100$ IU at $d_1 = 1$ cm, what is I_2 at $d_2 = 10$ cm?

$$100 \times 1^2 = I_2 \times 10^2 \quad \text{and}$$

$$I_2 = \frac{100 \times 1^2}{10^2} = \frac{100 \text{ IU}}{100} = 1 \text{ IU}$$

This is really a "work it in your head" problem. The distance increased by a factor of 10 so the intensity decreased by a factor of $10^2 = 100$. $100/100 = 1$.
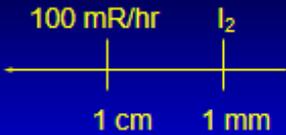
Here are several sample problems. Pay close attention to the units as they are often not the same!

Problem 1.

Problem 1. A reading of 100 mR/hr is obtained at a distance of 1 cm from a point source. What would be the reading at a distance of 1 mm?

Answer

Problem 1

100 mR/hr $\quad$ I_2

1 cm $\quad$ 1 mm

$100 \times 1^2 = I_2 \times 0.1^2$

$I_2 = 100 / 0.1^2 = 10{,}000$ mR/hr

Problem 2.

Problem 2. A reading of 287 mR/hr is obtained at a distance of 1.5 cm from a point source. What would be the reading at a distance of 15 cm?

Answer

Problem 2

287 mR/hr $\quad$ I_2

1.5 cm $\quad$ 15 cm

$287 \times 1.5^2 = I_2 \times 15^2$

$I_2 = 287 \times 1.5^2 / 15^2 = 2.87$ mR/hr

Keystrokes: $287 \times 1.5 \boxed{X^2} \div 15 \boxed{X^2} =$

Problem 3.

Problem 3. A reading of 287 mR/hr is obtained at a distance of 1.5 cm from a point source. At what distance would a reading of 28.7 mR/hr be obtained?

Answer

Problem 3

287 mR/hr   28.7

1.5 cm d_2

$287 \times 1.5^2 = 28.7 \times d_2^2$

$d_2 = \sqrt{287 \times 1.5^2 / 28.7} = \boxed{4.74 \text{ cm}}$

Keystrokes: $287 \times 1.5 \boxed{X^2} \div 28.7 = \boxed{\sqrt{}}$

Problem 4

Problem 4. A reading of 12 lumens/hr is obtained at a distance of 3.5 cm from a point source. At what distance would a reading of 1000 lumens/hr be obtained?

Answer

Problem 4

12 mR/hr   1,000

3.5 cm d_2

$12 \times 3.5^2 = 1000 \times d_2^2$

$d_2 = \sqrt{12 \times 3.5^2 / 1000} = \boxed{0.38 \text{ cm}}$

Keystrokes: $12 \times 3.5 \boxed{X^2} \div 1000 = \boxed{\sqrt{}}$

Problem 5

Problem 5. Denver is the mile-high city and it has been observed that the background radiation dose is approximately double that at sea level. The distance between sun and earth averages 93,000,000 miles. What impact does that one mile of distance have on the background radiation dose?

Answer: None

Problem 5

I_1 I_2

93,000,000 mi   92,999,999 mi

$$I_2 = I_1 \times \frac{(93,000,000)^2}{(92,999,999)^2} = 1.000,000,0215$$

impact of ISL: dose increases by 2 parts in 100 million!

Gamma Ray Dose Rate Constant

Γ represents the dose in rads delivered by a 1 mCi source held at a distance of 1 cm from a detector for 1 hr. It is expressed in units of

$$\frac{R \times cm^2}{hr \times mCi}$$

Problem 6

Problem 6. What would be the absorbed dose in Rads if a detector was positioned 5 cm from a 5 mCi source for 5 hours for the isotope whose gamma ray dose rate constant is 5 R·cm²/hr·mCi? (HINT: make sure all units cancel!)

Answer

If Γ is 5 R x cm²/hr x mCi, time = 5 hr, activity = 5 hr, and distance = 5 cm,

$$D_{Rads} = \frac{5\,R \times cm^2 \times 5\,hr \times 5\,mCi}{hr \times mCi \times 5^2\,cm^2}$$

$$D_{Rads} = 5\,R$$

You will note that the distance in the denominator, 5 cm, had to be squared. Aside from the fact that this is basically a complex inverse square law problem, if the 5 cm were not squared, the units would not have canceled.

The next problem is similar, except that exposure time was not given so the answer will be expressed in terms of dose rate (R/hr) rather than dose (R). In this next problem, pay close attention to the units!!

Problem 7

Problem 7. The gamma ray dose rate constant Γ for a particular isotope is $\underline{1.65\ R \times cm^2}$.

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ hr x mCi

If a 6 mCi source is held at a distance of 8 cm from the detector, what would be the dose rate in mR/hr?

a. 0.1547
b. 154.7
c. 1.238
d. 1238

Answer

Correct answer is "b".

$$\Gamma = \frac{1.65\ R \times cm^2}{hr \times mCi} \times \frac{6\ mCi}{8^2 cm^2} \quad \text{and}$$

Dose = 0.1547 R/hr = 154.7 mR/hr

This is basically an inverse square problem!